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High-electric-field quantum transport for semiconductor superlattices

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Abstract. Based on the Kadanoff–Baym–Keldysh non-equilibrium Green's function technique, a quantum transport theory for semiconductor superlattices under high-electric fields is developed. Both intra-collisional field effects and a collisional broadening are taken into account. The symmetry of the carrier system in an electric field and an extension of the generalized Kadanoff–Baym ansatz are discussed. The lifetime broadening of electron–phonon resonances due to scattering on impurities is calculated. Even if the mean impurity scattering strength is considerably smaller than the miniband width, the oscillatory current anomalies, which result from intra-collisional field effects, can be completely smeared out.

1. Introduction

Since the pioneering work by Esaki and Tsu [1], theoretical [2–11] and experimental [12–15] investigations have revealed many interesting properties of the superlattice (SL) transport such as negative differential conductivity (NDC) [12], Bloch oscillations [16] the formation of field domains [17], and even absolute negative current [18] under intense terahertz irradiation. The narrow wave vector minizone and energy bands of a SL allow electrons, accelerated perpendicular to the layers, to probe the negative-effective-mass region of the non-parabolic energy band giving rise to NDC. If the Bloch frequency $\Omega = eEd/\hbar$ (E is the electric field strength and d the SL period) is larger than some effective scattering rate $1/\tau$, carriers confined to the lowest miniband are expected to be Bragg reflected before being scattered by phonons or imperfections in the crystal. This gives rise to Bloch oscillations, the counterpart of which is the formation of a Wannier–Stark (WS) ladder in the energy domain. The electric field induced WS localization results in non-analytic resonant-type anomalies in the current–voltage characteristic (I–V) known as electron–phonon resonances. [19] For narrow band semiconductors, this non-monotonic I–V dependence has been studied both experimentally [20, 21] and theoretically [19, 22–24] many years ago. The anisotropic band structure of an artificial SL can be tuned over a wide range to establish optimal conditions for the observation of such an interesting quantum effect. Recently, Bloch oscillations and WS localization have been unambiguously identified in electro-optical experiments on SLs (for a review, see [25]). In contrast, there are no experiments that clearly demonstrate quantum effects in the SL miniband transport. To resolve this puzzle, it is necessary to develop a quantum transport theory that allows a study of electron–phonon resonances and includes scattering induced lifetime broadening to understand why the experimental identification of transport anomalies seems to be so difficult in a SL.

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Most previous theoretical work was based either on the quasi-classical Boltzmann [2, 3] or balance equations [7, 8], which completely neglected quantum WS localization under biasing. A quantum transport theory that reproduced most other approaches in this field has been recently proposed [26]. However, the fundamental difficulty of this approach is that inelastic scattering that dissipates the energy is completely neglected. As a consequence, current anomalies due to electron–phonon resonances could not be studied. This also refers to an extensive study [9] of instabilities of the electric field distribution that focused on quasi-elastic scattering on acoustic phonons. Other quantum mechanical treatments of the SL transport under strong dc bias without [11, 27] and with [28–30] an additional magnetic field considered the heating of the lateral electron motion and focused on current anomalies, which result from the WS localization. These anomalies are due to intra-collisional field effects (ICFEs). [11] However, in all these approaches the lifetime broadening of energetic states has been taken into account only via a phenomenological broadening parameter. A rigorous quantum transport theory of SLs that accounts for both a finite collisional broadening and the quantum WS localization is still absent. It is the aim of this paper to fill this gap by proposing a non-equilibrium Green's function theory that is capable of overcoming the above-mentioned limitations of former approaches.

The paper is organized as follows. In section 2, we introduce our notation and identify the main symmetry properties of correlation functions. As an application of this general formulation, we rederive in appendix A a quantum-kinetic equation that has been obtained many years ago using density matrix techniques [19, 31–33]. Particular emphasis is put on the symmetry of Green's functions and its impact on the Kadanoff–Baym (KB) ansatz. The mutual influence of both the collisional broadening and the WS localization on the SL transport is treated in section 3. The current density is calculated in section 4, and section 5 summarizes the main results of our paper.

2. Basic theory and symmetry properties

In this section, we will apply the non-equilibrium Green's function technique to review basic theoretical results and to discuss inherent symmetry properties of an electron system in a constant electric field. This part of our paper does not report completely new results (for an alternative formulation see, e.g. [33]), but collects all main ingredients that cover the physics of stationary quantum transport in nanostructures. As an application of this general approach, we will derive in appendix A a well-established quantum-kinetic equation [19, 31, 32] that describes ICFEs. We will restrict ourselves to intra-band processes induced by a homogeneous electric field and assume that the carriers remain essentially within the lowest energy band. A study of WS localization requires an exact treatment of electric field effects. This is achieved by including the electric field into the unperturbed part H_0 of the total Hamiltonian. Our calculation is based upon the Kadanoff–Baym–Keldysh non-equilibrium Green's function technique, [34, 35] where the symmetry properties of the correlation functions will play an important role in our calculation. Within the Keldysh formalism, the four double-time Green functions $G^{\lessgtr}$, G^c and $\tilde{G}^c$ (for their definition see [34, 36, 37]) and the related self-energies Σ can be arranged into two by two matrices

$$\hat{G} = \begin{pmatrix} G^c & -G^< \\ G^> & -\tilde{G}^c \end{pmatrix} \qquad \hat{\Sigma} = \begin{pmatrix} \Sigma^c & -\Sigma^< \\ \Sigma^> & -\tilde{\Sigma}^c \end{pmatrix} \tag{1}$$

the elements of which are not independent from each other, but satisfy the relationships $G^< + G^> = G^c + \tilde{G}^c$ and $\Sigma^< + \Sigma^> = \Sigma^c + \tilde{\Sigma}^c$. The equations of motion obeyed by

Green's functions are given by the Dyson equation, the matrix notation of which is

$$\left[\mathrm{i}\hbar\frac{\partial}{\partial t} - H_0(x)\right]\hat{G}(x,x') = \hbar\delta^4(x-x')\hat{I} + \hbar\int \mathrm{d}x''\,\hat{\Sigma}(x,x'')\hat{G}(x'',x'). \quad (2)$$

Here $x = (\boldsymbol{r}, t)$ denotes the space and time coordinates and $\hat{I}$ the unit matrix. The time integral runs over the upper and lower branch of the closed real-time contour [37, 38]. The values of Green's functions on the upper and lower time branch are related to each other by the symmetry property

$$G^{\gtrless}(x,x')^* = -G^{\gtrless}(x',x). \quad (3)$$

We will consider only weak scattering of carriers on phonons and impurities so that the self-consistent Born approximation can be used to calculate the self-energy. In this case, the self-energy components Σ^c and $\tilde{\Sigma}^c$ can be expressed by $\Sigma^{\lessgtr}$ via the equations

$$\Sigma^c(x,x') = \Theta(t-t')\Sigma^{>}(x,x') + \Theta(t'-t)\Sigma^{<}(x,x') \quad (4)$$

$$\tilde{\Sigma}^c(x,x') = \Theta(t'-t)\Sigma^{>}(x,x') + \Theta(t-t')\Sigma^{<}(x,x'). \quad (5)$$

The Dyson equation (2) is written down in the wavenumber representation by taking into account the definition of the correlation functions along the time contour. We get from equation (2)

$$\left[\mathrm{i}\hbar\frac{\partial}{\partial t} - \varepsilon(\boldsymbol{k}) + \mathrm{i}e\boldsymbol{E}\nabla_{\boldsymbol{k}}\right] G^{\gtrless}(\boldsymbol{k}t|\boldsymbol{k}'t') = \pm\hbar\int \mathrm{d}\boldsymbol{k}_1 \left\{ \int_{t'}^{t} \mathrm{d}t_1\,\Sigma^{\gtrless}(\boldsymbol{k}t|\boldsymbol{k}_1t_1)G^{\gtrless}(\boldsymbol{k}_1t_1|\boldsymbol{k}'t') + \int_{-\infty}^{t'} \mathrm{d}t_1\,\Sigma^{\gtrless}(\boldsymbol{k}t|\boldsymbol{k}_1t_1)G^{\lessgtr}(\boldsymbol{k}_1t_1|\boldsymbol{k}'t') - \int_{-\infty}^{t} \mathrm{d}t_1\,\Sigma^{\lessgtr}(\boldsymbol{k}t|\boldsymbol{k}_1t_1)G^{\gtrless}(\boldsymbol{k}_1t_1|\boldsymbol{k}'t') \right\} \quad (6)$$

where $\varepsilon(\boldsymbol{k})$ is the energy dispersion relation. The Dyson equation (6) simplifies further, when the symmetries of Green's functions and self-energies are exploited. For stationary carrier transport and for the considered scalar potential gauge, Green's functions depend on two wavenumber vectors and only on the time difference [39]

$$G^{\gtrless}(\boldsymbol{k}t|\boldsymbol{k}'t') = G^{\gtrless}(\boldsymbol{k},\boldsymbol{k}'|t'-t). \quad (7)$$

Accordingly, the symmetry relation (3) translates into

$$G^{\gtrless}(\boldsymbol{k},\boldsymbol{k}'|t)^* = -G^{\gtrless}(\boldsymbol{k}',\boldsymbol{k}|-t). \quad (8)$$

A further simplification of the Dyson equation (6) relies on a spatial symmetry property of Green's functions that comes into play, although the carrier system under the influence of an external electric field is no longer translationally invariant. Such a symmetry, associated with the translation operator, reflects the fact that, when an electron moves to a point $\boldsymbol{r}$ under the influence of the field $\boldsymbol{E}$, the momentum is restored, if the energy is shifted by $e\boldsymbol{E}\boldsymbol{r}$. This symmetry implies (cf equation (17) in [40])

$$G^{\gtrless}(\boldsymbol{k},\boldsymbol{k}'|t) = G^{\gtrless}(\boldsymbol{k},t)\delta\left(\boldsymbol{k}' - \boldsymbol{k} - \frac{e\boldsymbol{E}}{\hbar}t\right). \quad (9)$$

The same relationship also applies to the self-energy. The symmetry relation in equation (8) together with equation (9) becomes particularly transparent for Green's functions defined by

$$\tilde{G}^{\gtrless}(\boldsymbol{k},t) \equiv G^{\gtrless}\left(\boldsymbol{k} - \frac{e\boldsymbol{E}}{2\hbar}t, t\right) \quad (10)$$

for which we get

$$\tilde{G}^{\gtrless}(\boldsymbol{k},t)^* = -\tilde{G}^{\gtrless}(\boldsymbol{k},-t). \quad (11)$$

The main step in almost all studies of quantum transport is to rewrite the set of Dyson equations by means of the KB ansatz to obtain a closed equation for the time diagonal distribution function. This can be achieved by introducing new functions $R^{\lessgtr}$

$$\tilde{G}^{\gtrless}(\boldsymbol{k},t) = \mp \mathrm{i} G(\boldsymbol{k},t) R^{\gtrless}(\boldsymbol{k},t) \tag{12}$$

with

$$G(\boldsymbol{k},t) = \exp\left[\frac{\mathrm{i}}{\hbar}\int_{-t/2}^{t/2} \mathrm{d}\tau\, \varepsilon\left(\boldsymbol{k} + \frac{e\boldsymbol{E}}{\hbar}\tau\right)\right]. \tag{13}$$

According to equation (11), the functions $R^{\lessgtr}$ must fulfil the symmetry relation

$$R^{\gtrless}(\boldsymbol{k},t) = R^{\gtrless}(\boldsymbol{k},-t)^{*} \tag{14}$$

and are the solutions of the differential equations

$$\left(\frac{\partial}{\partial t} \pm \frac{e\boldsymbol{E}}{2\hbar}\nabla_{k}\right) R^{\gtrless}(\boldsymbol{k},t) = 0 \tag{15}$$

when scattering is completely neglected (cf equation (45) in appendix A). A solution of this equation that is in accordance with the symmetry relation (14) has the form

$$R^{\gtrless}(\boldsymbol{k},t) = f^{\gtrless}\left(\boldsymbol{k} - \frac{e\boldsymbol{E}}{2\hbar}|t|\right) \tag{16}$$

where $f^{\lessgtr}$ are unknown functions, which according to the initial condition (cf equation (40) in appendix A) are related to each other by

$$f^{<}(\boldsymbol{k}) + f^{>}(\boldsymbol{k}) = 1. \tag{17}$$

Equations (12) and (16) solve the kinetic equation even in the more general case, when scattering is taken into account, provided lifetime broadening effects are neglected in the self-energy expression. These equations, known as the generalized KB ansatz [41, 42], have been used in the literature to study quantum transport in semiconductors. Unlike the conventional KB ansatz, which has fundamental limitations, this new ansatz is fully consistent with the dynamical structure of the theory and agrees exactly with results derived from the Liouville equation for the density matrix [41]. This ansatz takes the causality for the time evolution of the particle propagator properly into account and follows unambiguously from the symmetry properties of the electron system in an external electric field. If, however, lifetime broadening becomes important, the ansatz (16) no longer solves the kinetic equation, and one has to determine a distribution function $f^{\gtrless}(\boldsymbol{k}-e\boldsymbol{E}|t|/2\hbar, t)$ that depends explicitly on a time variable even for stationary transport problems. This leads to additional complications, because a closed equation cannot be derived for the distribution function $f^{\gtrless}(\boldsymbol{k},0)$, which is used to calculate the current.

3. Treatment of ICFEs and collisional broadening

In this section, we treat high-field transport in SLs under the condition of low carrier concentration so that field domain formation is suppressed and the carriers approximately obey the Boltzmann statistics. In this case, the Dyson equation for $G^{\lessgtr}$ simplifies considerably because one can treat $G^{<}$ by a perturbational method. This allows us to decouple the set of equations for $G^{<}$ and $G^{>}$ by exploiting an equation of the form $G^{<}(\boldsymbol{k},\omega) = -G^{>}(\boldsymbol{k},\omega) f(\boldsymbol{k})$, which holds true for an equilibrium electron system. If lifetime broadening effects become essential, the eigenenergies of the system are no longer sharp, which implies that the electron distribution function depends explicitly on a time variable as has been stressed at the end of the last section. This explicit time dependence, which expresses the non-Markovian character of the transport, becomes smooth or nearly disappears when collisional broadening plays a minor role. For all other cases, the explicit time dependence of the distribution function must be retained. Therefore, we employ the following ansatz for a non-degenerate electron gas:

$$\tilde{G}^{<}(\boldsymbol{k},t) = -\tilde{G}^{>}(\boldsymbol{k},t) f\left(\boldsymbol{k} - \frac{e\boldsymbol{E}}{2\hbar}|t|, t\right) \tag{18}$$

where the distribution functions $f(\boldsymbol{k},t)$ and $\tilde{G}^{>}(\boldsymbol{k},t)$ are determined from quite different equations. First, we will derive an equation for Green's function $\tilde{G}^{>}$. This function is calculated from the Dyson equation (cf equation (39) in appendix A) under the condition $G^{<} \to 0$. We obtain

$$\left[\mathrm{i}\hbar\frac{\partial}{\partial t} + \varepsilon\left(\boldsymbol{k} - \frac{e\boldsymbol{E}}{2\hbar}t\right)\right]\overline{G}(\boldsymbol{k},t) = \hbar\int_0^t \mathrm{d}t_1\,\overline{\Sigma}\left(\boldsymbol{k} - \frac{e\boldsymbol{E}}{2\hbar}t_1, t-t_1\right)\overline{G}(\boldsymbol{k},t_1) \tag{19}$$

where the following Green's function and self-energy have been introduced:

$$\overline{G}(\boldsymbol{k},t) = G^{>}\left(\boldsymbol{k} - \frac{e\boldsymbol{E}}{\hbar}t, t\right) \qquad \overline{\Sigma}(\boldsymbol{k},t) = \Sigma^{>}\left(\boldsymbol{k} - \frac{e\boldsymbol{E}}{\hbar}t, t\right) \tag{20}$$

and where according to equation (40) in appendix A the initial condition $\overline{G}(\boldsymbol{k},0) = -\mathrm{i}$ has to be fulfilled. This suggests the need to search for a solution of equation (19) in the form

$$\overline{G}(\boldsymbol{k},t) = -\mathrm{i}\exp\left[\frac{\mathrm{i}}{\hbar}\int_0^t \mathrm{d}\tau\, h(\boldsymbol{k},\tau)\right]. \tag{21}$$

From equations (19) and (21), we get

$$h(\boldsymbol{k},t) = \varepsilon\left(\boldsymbol{k} - \frac{e\boldsymbol{E}}{2\hbar}t\right) - \hbar\int_0^t \mathrm{d}t_1\overline{\Sigma}\left(\boldsymbol{k} - \frac{e\boldsymbol{E}}{2\hbar}t_1, t-t_1\right)\exp\left[\frac{\mathrm{i}}{\hbar}\int_t^{t_1}\mathrm{d}\tau\, h(\boldsymbol{k},\tau)\right]. \tag{22}$$

This integral equation allows the calculation of the unknown function $h(\boldsymbol{k},t)$, which must satisfy the symmetry relation

$$\varphi(\boldsymbol{k},t) \equiv \int_0^t \mathrm{d}\tau\, h(\boldsymbol{k},\tau) = \int_0^t \mathrm{d}\tau\, h\left(\boldsymbol{k} - \frac{e\boldsymbol{E}}{2\hbar}t, -\tau\right)^* \tag{23}$$

as can be seen from its definition. Specific results will be derived for a SL with a simple tight-binding energy band

$$\varepsilon(\boldsymbol{k}) = \varepsilon(\boldsymbol{k}_\perp) + \varepsilon(k_z) = \frac{\hbar^2\boldsymbol{k}_\perp^2}{2m^*} + \frac{\Delta}{2}[1-\cos(k_z d)] \tag{24}$$

where m^* denotes the effective mass of the lateral electron motion and Δ the miniband width. We will include lifetime effects due to scattering of electrons on impurities and retain only the first-order correction with respect to scattering in equation (22) (i.e. $h(\boldsymbol{k},\tau)$ is replaced by $\varepsilon(\boldsymbol{k}-e\boldsymbol{E}\tau/2\hbar)$ on the right-hand side of equation (22)). The complex self-energy, which enters equation (22), renormalizes the energy band (this is assumed to be accomplished already in equation (22)) and leads to a finite lifetime of the electronic states. The collisional broadening depends on the electric field and is composed of a smooth and a strongly oscillating part. Only the smooth contribution, which we will calculate within the quasi-classical approximation [11] ($\Delta/\hbar\Omega > 1$), is expected to determine the main lifetime effects. In the quasi-classical limit, the scattering induced damping is independent of the electric field. When $|t| \to \infty$, we get from equations (22), (23), and (43) in appendix A the following impurity mediated lifetime broadening:

$$\varphi_{im}(\boldsymbol{k},t) = -s(\varepsilon(\boldsymbol{k}_\perp))|t| \tag{25}$$

where

$$s(\varepsilon(\boldsymbol{k}_\perp)) = \frac{u^2}{d}\int \frac{\mathrm{d}^2\boldsymbol{k}'_\perp}{(2\pi)^2}\mathrm{Re}\int_0^\infty \mathrm{d}t\, \exp\left[\frac{\mathrm{i}}{\hbar}(\varepsilon(\boldsymbol{k}'_\perp) - \varepsilon(\boldsymbol{k}_\perp))t\right] J_0^2\left(\frac{\Delta}{2\hbar}t\right). \tag{26}$$

J_0 is the Bessel function. The damping function $s(\varepsilon(\boldsymbol{k}_\perp))$ is always positive and depends only on the lateral wavevector $\boldsymbol{k}_\perp$ via $\varepsilon(\boldsymbol{k}_\perp)$. Introducing the area density of states by

$$\rho_\perp(\varepsilon) = \int \frac{\mathrm{d}^2\boldsymbol{k}_\perp}{(2\pi)^2}\delta(\varepsilon - \varepsilon(\boldsymbol{k}_\perp)) = \frac{m^*}{2\pi\hbar^2}\Theta(\varepsilon) \tag{27}$$

we get from equation (26) the following final result for the energy dependent collision frequency:

$$s(\varepsilon) = \frac{m^* u^2}{\pi^2\hbar\Delta d}\int_{\max(0,\varepsilon-\Delta)}^{\varepsilon+\Delta} \mathrm{d}\varepsilon'\, K\left(\sqrt{1 - \left(\frac{\varepsilon - \varepsilon'}{\Delta}\right)^2}\right) \tag{28}$$

where K is the complete elliptic integral of the first kind. The next step is the derivation of a kinetic equation for the distribution function f, which enters the Dyson equation via the Green's function $\tilde{G}^<$ in equation (18). The factor $\tilde{G}^>$, which enters this equation, too, has just been calculated. We obtain from equations (20), (21) and (25)

$$\tilde{G}^>(\boldsymbol{k}, t) = -\mathrm{i}\exp\left[\frac{\mathrm{i}}{\hbar}\int_{-t/2}^{t/2}\mathrm{d}\tau\, \varepsilon\left(\boldsymbol{k} - \frac{e\boldsymbol{E}}{\hbar}\tau\right) - s(\varepsilon(\boldsymbol{k}_\perp))|t|\right]. \tag{29}$$

It remains to derive a kinetic equation for $f(\boldsymbol{k}, t)$. Even for a non-degenerate electron gas, this results in a complicated integral equation, the explicit form of which is given in appendix B.

Our paper is mainly concerned with the study of electron–phonon resonances in the SL transport and their damping due to collisional broadening. As shown in section 4, these resonances survive only when the lifetime broadening is extremely small. In this case, $f(\boldsymbol{k}, t)$ does not depend explicitly on t, and the kinetic equation simplifies accordingly. From equation (51) in appendix B together with equations (20), (21), and (25) we get

$$\frac{e}{\hbar}\boldsymbol{E}\nabla_{\boldsymbol{k}} f(\boldsymbol{k}) = \sum_{\boldsymbol{k}'} W(\boldsymbol{k}, \boldsymbol{k}') f(\boldsymbol{k}') \tag{30}$$

where the field dependent scattering probability is given by

$$\begin{aligned} W(\boldsymbol{k}, \boldsymbol{k}') = \frac{2}{\hbar^2}\mathrm{Re}\int_0^\infty \mathrm{d}t \sum_{\boldsymbol{q}\lambda} |M_{\boldsymbol{q}\lambda}|^2[(N_{\boldsymbol{q}\lambda} + 1)\mathrm{e}^{-\mathrm{i}\omega_{\boldsymbol{q}\lambda}t} + N_{\boldsymbol{q}\lambda}\mathrm{e}^{\mathrm{i}\omega_{\boldsymbol{q}\lambda}t}] \\ \times\left[P\left(\boldsymbol{k}' + \frac{\boldsymbol{q}}{2}, \boldsymbol{k} - \frac{\boldsymbol{q}}{2}, \boldsymbol{q}|t\right) - P\left(\boldsymbol{k}' + \frac{\boldsymbol{q}}{2}, \boldsymbol{k} + \frac{\boldsymbol{q}}{2}, \boldsymbol{q}|t\right)\right] \end{aligned} \tag{31}$$

and where we introduced the correlation functions

$$\begin{aligned} P(\boldsymbol{k}', \boldsymbol{k}, \boldsymbol{q}|t) = \exp\left\{-\frac{\mathrm{i}}{\hbar}\int_0^t \mathrm{d}\tau\, \left(\varepsilon\left(\boldsymbol{k}' + \frac{\boldsymbol{q}}{2} + \frac{e\boldsymbol{E}}{\hbar}\tau\right) - \varepsilon\left(\boldsymbol{k}' - \frac{\boldsymbol{q}}{2} + \frac{e\boldsymbol{E}}{\hbar}\tau\right)\right)\right\} \\ \times\delta_{\boldsymbol{k}'+e\boldsymbol{E}t/\hbar,\boldsymbol{k}}\exp\left\{-ts\left(\varepsilon\left(\boldsymbol{k}'_\perp + \frac{\boldsymbol{q}_\perp}{2}\right)\right) - ts\left(\varepsilon\left(\boldsymbol{k}'_\perp - \frac{\boldsymbol{q}_\perp}{2}\right)\right)\right\}. \end{aligned} \tag{32}$$

$N_{\boldsymbol{q}\lambda}$ denotes the Bose–Einstein distribution function for phonons. Equation (30) extends former theoretical results [11,19,43] by taking into account both ICFEs and finite lifetime effects within a quantum approach. The collisional broadening is manifest in an energy dependent scattering time $1/s(\varepsilon)$. The integro-differential equation (30) is solved together with the normalization condition for the distribution function $f(\boldsymbol{k})$.

4. Calculation of the current density

The calculation of the stationary current density requires the determination of the time dependent electron distribution function $f(\boldsymbol{k}, t)$ from the kinetic equation. In the case of weak lifetime broadening, there is only a weak explicit t dependence of the distribution function. A broadened Lorentzian energy conservation is obtained, when this weak time dependence is neglected. This approximation has the defect that higher and higher energy states become populated because the Lorentz curve falls off only gradually. To avoid this run-away effect, the explicit time dependence of the distribution function has to be retained. In our present analytic study, however, we will not address the details of such an analysis, but present some numerical results that reveal the main features of ICFEs and collisional broadening in the SL transport.

The current density is calculated from the stationary electron distribution function via

$$j_z = -\frac{e}{\hbar V}\sum_{\boldsymbol{k}} \varepsilon(k_z)\frac{\partial f(\boldsymbol{k})}{\partial k_z} \tag{33}$$

where V is the volume of the crystal. This equation has been derived by integration by parts and relates the current directly to the collision integral, when $\partial f(\boldsymbol{k}, 0)/\partial k_z$ is replaced by the right-hand side of the kinetic equation (30). To simplify the calculation, we will neglect the $\boldsymbol{q}$ dependence of the electron–phonon coupling and treat dispersionless polar–optical phonons

$$|M_{\boldsymbol{q}\lambda}|^2 \to \omega_0^2\Gamma \tag{34}$$

where Γ is an averaged coupling constant. In this case all wavenumber integrals can be calculated analytically. Electron–phonon resonances are predicted to appear at comparatively high electric fields, when NDC occurs. In the case of high electric fields and weak scattering ($\Omega\tau > 1$), the distribution function $f(\boldsymbol{k})$ can be replaced in a perturbational sense [19] by its lateral part $f(\boldsymbol{k}_\perp) = \sum_{k_z} f(\boldsymbol{k})$ on the right-hand side of the kinetic equation (30). This allows us to express the current density by the lateral distribution function $f(\boldsymbol{k}_\perp)$ and the scattering probability W via equations (30) and (33). From equations (27), (30)–(32) and (33) we get

$$j_z = \frac{2\omega_0^2\Gamma}{\hbar^2 E d^2}\left(\frac{m^*}{2\pi\hbar^2}\right)^2 \mathrm{Re}\int_0^\infty \mathrm{d}\varepsilon\,\mathrm{d}\varepsilon'\, f(\varepsilon')\int_0^\infty \mathrm{d}t\,\mathrm{e}^{-ts(\varepsilon)-ts(\varepsilon')+\mathrm{i}t(\varepsilon'-\varepsilon)/\hbar}$$
$$\times[(N_0+1)\mathrm{e}^{-\mathrm{i}\omega_0 t} + N_0\mathrm{e}^{\mathrm{i}\omega_0 t}]\frac{d}{2\pi}\int_0^{2\pi/d}\mathrm{d}q_z\, H(q_z, t) \tag{35}$$

where the function $H(q_z, t)$, which introduces ICFEs and the Stark ladder, is calculated from [44]

$$H(q_z, t) = \frac{d}{2\pi}\int_0^{2\pi/d}\mathrm{d}k_z\left[\varepsilon\left(k_z - \frac{q_z}{2}\right) - \varepsilon\left(k_z + \frac{q_z}{2}\right)\right]$$
$$\times\exp\left\{-\frac{\mathrm{i}}{\hbar}\int_0^t \mathrm{d}\tau\left[\varepsilon\left(k_z + \frac{q_z}{2} - \frac{eE}{\hbar}\tau\right) - \varepsilon\left(k_z - \frac{q_z}{2} - \frac{eE}{\hbar}\tau\right)\right]\right\}$$
$$= \sum_{l=-\infty}^{\infty}\mathrm{e}^{\mathrm{i}l\Omega t}\, l\hbar\Omega J_l^2\left(\frac{\Delta}{\hbar\Omega}\sin\frac{q_z d}{2}\right). \tag{36}$$

In this equation, J_l denotes the Bessel function of integer order l. In our study of electron–phonon resonances, we will not take into account the heating of the lateral electron motion by the electric field, i.e. we replace the function $f(\varepsilon)$ in equation (35) by the Boltzmann distribution ($\sim \exp(-\varepsilon/k_B T)$). This approximation allows us to grasp the main physics of the high-field SL transport. However, quantitative details can be derived only, when the quantum-kinetic equation is solved for the distribution function.

Inserting $H(q_z, t)$ from equation (36) into equation (35) and calculating the t integral, we obtain for the current density of a non-degenerate electron gas

$$j_z = \frac{em^* n_s \omega_0^2 \Gamma}{2\pi\hbar^4 k_B T d} \frac{1}{1-\mathrm{e}^{-\beta}} \sum_{l=-\infty}^{\infty} l F_l\left(\frac{\Delta}{\hbar\Omega}\right) \int_0^\infty \mathrm{d}\varepsilon\, \mathrm{d}\varepsilon'\, \mathrm{e}^{-\varepsilon'/k_B T} \times \left[\frac{s(\varepsilon)+s(\varepsilon')}{(l\Omega + (\varepsilon'-\varepsilon)/\hbar - \omega_0)^2 + (s(\varepsilon)+s(\varepsilon'))^2} + \mathrm{e}^{-\beta} \frac{s(\varepsilon)+s(\varepsilon')}{(l\Omega + (\varepsilon'-\varepsilon)/\hbar + \omega_0)^2 + (s(\varepsilon)+s(\varepsilon'))^2} \right] \tag{37}$$

where

$$F_l\left(\frac{\Delta}{\hbar\Omega}\right) = \frac{1}{\pi}\int_0^\pi \mathrm{d}z\, J_l^2\left(\frac{\Delta}{\hbar\Omega}\sin z\right). \tag{38}$$

We used the abbreviation $\beta = \hbar\omega_0/k_B T$. It is seen from equation (37) that current maxima are expected to appear at electron–phonon resonance positions determined from $l\Omega \pm \omega_0 = 0$. These resonances are broadened by the energy dependent scattering time $1/s(\varepsilon)$. Former approaches [28, 30, 43] that treated collisional broadening on a phenomenological level introduced a constant scattering time parameter ($s(\varepsilon) + s(\varepsilon') \to s$).

Numerical results calculated from equations (28) and (37) are shown in figures 1(*a*) and (*b*) for $\Delta/\hbar\omega_0 = 1$ and 0.5, respectively. The field independent reference current density is $j_{z0} = em^* n_s \omega_0^2 \Gamma / 2\pi\hbar^3 d$. As our restriction to the zeros Fourier component of the distribution function in the collision integral ($f(\boldsymbol{k}) \to f(\boldsymbol{k}_\perp) = \sum_{k_z} f(\boldsymbol{k})$) is valid only at high electric field strengths beyond the Ohmic regime, we calculated the current density only above 5 kV cm^{-1}. Vertical lines in figures 1(*a*) and (*b*) mark the positions of electron–phonon resonances at $E = \hbar\omega_0/led$ with $l = 1, 2, 3$ (for d we used 10 nm). The solid curves have been calculated for the case, when the impurity scattering strength is much smaller than the miniband width ($m^* u^2/\pi^2 \Delta d = 0.005$). In this case weak current oscillations appear, which, however, are rapidly smeared out, when the impurity strength becomes slightly larger. This is shown by the dashed curves, which have been calculated for $m^* u^2/\pi^2 \Delta d = 0.05$. The lifetime broadening effect calculated from the microscopic model seems to be larger than phenomenological estimates suggest. This is due to the fact that in equation (37) both energy integrals are affected by the damping function $s(\varepsilon)$. Another interesting result of our calculation is that the current density increases with increasing lifetime broadening below a field strength of about 30 kV cm^{-1}. This enhancement of the current is due to the fact that the WS localized electronic states become more and more delocalized, when the collisional broadening increases. Only at very high electric field strengths, when the electrons are strongly localized along the field direction, this effect is not that important.

Our calculation demonstrates that quantum effects in the SL miniband transport can appear only when the collisional broadening is drastically reduced. We believe that this is the reason why electron–phonon resonances have not been reported in experiments on SLs until now.

5. Summary

In summary, we have developed a quantum transport theory for semiconductor SLs under high electric fields. Particular attention has been paid to the symmetry properties of Green's functions. We stressed the known fact (see, e.g. [33]) that only the generalized KB ansatz is compatible with the symmetry of the electron system in an electric field. A proper treatment of collisional broadening requires us, however, to extend the generalized KB ansatz by introducing

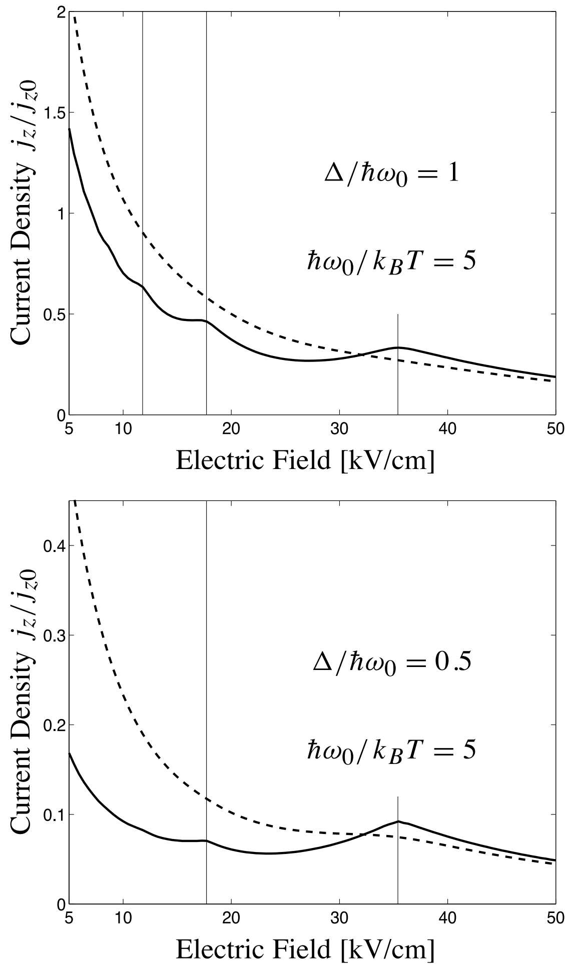

Figure 1. (*a*) Field dependence of the dimensionless current density j_z/j_{z0} with $j_{z0} = em^*n_s\omega^2\Gamma/2\pi\bar{h}^3d$ for $\Delta/\bar{h}\omega_0 = 1$ and $\bar{h}\omega_0/k_BT = 5$ calculated from equations (49), (50) and (40). The scattering strength parameter $m^*u^2/\pi^2\Delta d$ is given by 0.005 and 0.05 for the solid and dashed line, respectively. (*b*) The same as in (*a*) for $\Delta/\bar{h}\omega_0 = 0.5$.

a distribution function, which depends explicitly on a time variable, even for stationary transport problems.

We treated ICFEs and collisional broadening in an approximate manner to study current anomalies in the SL miniband transport associated with electron–phonon resonances that result from WS localization and their dependence on lifetime broadening. As an example, we treated collisional broadening due to elastic scattering on impurities and obtained the result that under the condition of high field transport the smooth part of the scattering time depends on the energy of the lateral electron motion. Our numerical results demonstrate that current oscillations, which are due to ICFEs, occur only when the lifetime broadening is extremely small. This might be the reason why quantum mechanical current oscillations have not been reported in high-field transport measurements in SLs up to now.

One interesting extension of our approach would be the study of Zener tunnelling within a multiple sub-band SL model. Recently, very exciting experimental results have been published [47] in this field. One technical, but not fundamental difficulty in such a theory is the calculation of off-diagonal elements of the density matrix with respect to the band index.

Finally, we hope that our quantum-mechanical approach, which accounts for both ICFEs and collision broadening, can be also used to study the stationary transport in other nanostructure devices.

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Appendix A

In this appendix the symmetry properties discussed in section 2 are exploited to derive a kinetic equation that plays a central role in the treatment of quantum transport.

Our starting point is the Dyson equation (6), in which equations (7) and (9) are inserted and $t' - t$ is replaced by t. We get

$$\left[-\mathrm{i}\hbar\frac{\partial}{\partial t} - \varepsilon(\boldsymbol{k}) + \mathrm{i}e\boldsymbol{E}\nabla_{\boldsymbol{k}}\right] G^{\gtrless}(\boldsymbol{k},t) = \pm\hbar\Bigg\{ -\int_0^t \mathrm{d}t_1\, \Sigma^{\gtrless}(\boldsymbol{k},t_1) G^{\gtrless}\left(\boldsymbol{k} + \frac{e\boldsymbol{E}}{\hbar}t_1, t-t_1\right) + \int_0^\infty \mathrm{d}t_1\, \Sigma^{\gtrless}(\boldsymbol{k},t-t_1) G^{\lesseqgtr}\left(\boldsymbol{k} + \frac{e\boldsymbol{E}}{\hbar}(t-t_1), t_1\right) - \int_{-\infty}^0 \mathrm{d}t_1\, \Sigma^{\lesseqgtr}(\boldsymbol{k},t_1) G^{\gtrless}\left(\boldsymbol{k} + \frac{e\boldsymbol{E}}{\hbar}t_1, t-t_1\right)\Bigg\} \tag{39}$$

where the initial condition

$$G^{>}(\boldsymbol{k},0) - G^{<}(\boldsymbol{k},0) = -\mathrm{i} \tag{40}$$

must be fulfilled. According to equation (8), the solution of this integro-differential equation has to satisfy the symmetry relation

$$G^{\gtrless}(\boldsymbol{k},t)^* = -G^{\gtrless}\left(\boldsymbol{k} + \frac{e\boldsymbol{E}}{\hbar}t, -t\right). \tag{41}$$

We will treat scattering within the self-consistent Born approximation, where the self-energy is calculated from

$$\Sigma^{\gtrless}(\boldsymbol{k},t) = \mathrm{i}\int \frac{\mathrm{d}\boldsymbol{k}'}{(2\pi)^3} D^{\gtrless}(\boldsymbol{k}-\boldsymbol{k}',t) G^{\gtrless}(\boldsymbol{k}',t). \tag{42}$$

For scattering on impurities the Born approximation yields

$$\Sigma_{im}^{\gtrless}(\boldsymbol{k},t) = u^2 \int \frac{\mathrm{d}\boldsymbol{k}'}{(2\pi)^3} G^{\gtrless}(\boldsymbol{k}',t) \tag{43}$$

where u^2 is the strength of the impurity potential. For simplicity we will restrict our consideration of the electron–phonon interaction to the bulk phonon model. In this case the self-energy is given by [33, 34]

$$\Sigma_{ph}^{\gtrless}(\boldsymbol{k},t) = \frac{2\pi}{\hbar^2}\sum_{\boldsymbol{q}\lambda} \frac{|M_{\boldsymbol{q}\lambda}|^2}{\sinh(\hbar\omega_{\boldsymbol{q}\lambda}/2k_B T)} \cos\omega_{\boldsymbol{q}\lambda}\left(t \mp \frac{\mathrm{i}\hbar}{2k_B T}\right) G^{\gtrless}(\boldsymbol{k}+\boldsymbol{q},t) \tag{44}$$

where $\omega_{\boldsymbol{q}\lambda}$ is the phonon frequency of wavevector $\boldsymbol{q}$ in branch λ and $M_{\boldsymbol{q}\lambda}$ the screened electron–phonon coupling matrix element. In equation (44), T is the temperature and k_B the Boltzmann constant. The Dyson equation (39) together with equation (43) or (44) form a closed system of two coupled equations, from which $G^{<}$ and $G^{>}$ can be calculated.

Now we will derive a kinetic equation for Green's function $\tilde{G}^{\gtrless}(\boldsymbol{k}, t)$ defined in equation (10). Such an equation results from the difference of Dyson equations written down for $\tilde{G}^{\gtrless}(\boldsymbol{k}, t)$ and $\tilde{G}^{\gtrless}(\boldsymbol{k}, -t)^*$, respectively. After elementary calculations we arrive at the following non-Markovian integral equations for the correlation functions

$$\left[\varepsilon\left(\boldsymbol{k}+\frac{e\boldsymbol{E}}{2\hbar}t\right)-\varepsilon\left(\boldsymbol{k}-\frac{e\boldsymbol{E}}{2\hbar}t\right)+\mathrm{i}e\boldsymbol{E}\boldsymbol{\nabla}_{\boldsymbol{k}}\right]\tilde{G}^{\gtrless}(\boldsymbol{k}, t)$$
$$= \pm\hbar\left\{-\int_0^t \mathrm{d}t_1\left[\tilde{\Sigma}^{\gtrless}\left(\boldsymbol{k}-\frac{e\boldsymbol{E}}{2\hbar}(t-t_1), t_1\right)\tilde{G}^{\gtrless}\left(\boldsymbol{k}+\frac{e\boldsymbol{E}}{2\hbar}t_1, t-t_1\right)\right.\right.$$
$$\left.-\tilde{\Sigma}^{\gtrless}\left(\boldsymbol{k}+\frac{e\boldsymbol{E}}{2\hbar}(t-t_1), t_1\right)\tilde{G}^{\gtrless}\left(\boldsymbol{k}-\frac{e\boldsymbol{E}}{2\hbar}t_1, t-t_1\right)\right]$$
$$+\int_0^\infty \mathrm{d}t_1\left[\tilde{\Sigma}^{\gtrless}\left(\boldsymbol{k}-\frac{e\boldsymbol{E}}{2\hbar}t_1, t-t_1\right)\tilde{G}^{\lessgtr}\left(\boldsymbol{k}+\frac{e\boldsymbol{E}}{2\hbar}(t-t_1), t_1\right)\right.$$
$$\left.-\tilde{\Sigma}^{\lessgtr}\left(\boldsymbol{k}+\frac{e\boldsymbol{E}}{2\hbar}(t-t_1), t_1\right)\tilde{G}^{\gtrless}\left(\boldsymbol{k}-\frac{e\boldsymbol{E}}{2\hbar}t_1, t-t_1\right)\right]$$
$$-\int_{-\infty}^0 \mathrm{d}t_1\left[\tilde{\Sigma}^{\lessgtr}\left(\boldsymbol{k}-\frac{e\boldsymbol{E}}{2\hbar}(t-t_1), t_1\right)\tilde{G}^{\gtrless}\left(\boldsymbol{k}+\frac{e\boldsymbol{E}}{2\hbar}t_1, t-t_1\right)\right.$$
$$\left.\left.-\tilde{\Sigma}^{\gtrless}\left(\boldsymbol{k}+\frac{e\boldsymbol{E}}{2\hbar}t_1, t-t_1\right)\tilde{G}^{\lessgtr}\left(\boldsymbol{k}-\frac{e\boldsymbol{E}}{2\hbar}(t-t_1), t_1\right)\right]\right\}. \tag{45}$$

In the stationary case, the current density is calculated from Green's function $\tilde{G}^{<}(\boldsymbol{k}, 0)$ that satisfies the equation

$$\mathrm{i}e\boldsymbol{E}\nabla_k\tilde{G}^{\gtrless}(\boldsymbol{k}, 0) = \mp 2\hbar\mathrm{Re}\int_0^\infty \mathrm{d}t_1\left\{\tilde{\Sigma}^{\gtrless}\left(\boldsymbol{k}-\frac{e\boldsymbol{E}}{2\hbar}t_1, t_1\right)^*\tilde{G}^{\lessgtr}\left(\boldsymbol{k}-\frac{e\boldsymbol{E}}{2\hbar}t_1, t_1\right)\right.$$
$$\left.-\tilde{\Sigma}^{\lessgtr}\left(\boldsymbol{k}-\frac{e\boldsymbol{E}}{2\hbar}t_1, t_1\right)^*\tilde{G}^{\gtrless}\left(\boldsymbol{k}-\frac{e\boldsymbol{E}}{2\hbar}t_1, t_1\right)\right\} \tag{46}$$

which is not a closed one for $\tilde{G}^{<}(\boldsymbol{k}, 0)$. If the influence of the electric field on scattering (ICFE) is taken into account, but collisional broadening is neglected, a closed kinetic equation for the distribution function is obtained by making use of the generalized KB ansatz (equations (12) and (16)). Inserting this ansatz into equation (46), we get

$$e\boldsymbol{E}\nabla_k f^{\gtrless}(\boldsymbol{k}) = \mp 2\hbar\mathrm{Re}\,\mathrm{i}\int_0^\infty \mathrm{d}t\, g(\boldsymbol{k}, t)\left\{\tilde{\Sigma}^{\gtrless}\left(\boldsymbol{k}-\frac{e\boldsymbol{E}}{2\hbar}t, t\right)^* f^{\lessgtr}\left(\boldsymbol{k}-\frac{e\boldsymbol{E}}{\hbar}t\right)\right.$$
$$\left.+\tilde{\Sigma}^{\lessgtr}\left(\boldsymbol{k}-\frac{e\boldsymbol{E}}{2\hbar}t, t\right)^* f^{\gtrless}\left(\boldsymbol{k}-\frac{e\boldsymbol{E}}{\hbar}t\right)\right\} \tag{47}$$

where

$$g(\boldsymbol{k}, t) = \exp\left[\frac{\mathrm{i}}{\hbar}\int_0^t \mathrm{d}\tau\,\varepsilon\left(\boldsymbol{k}-\frac{e\boldsymbol{E}}{\hbar}\tau\right)\right]. \tag{48}$$

If scattering on polar optical phonons is the main inelastic scattering mechanism, it is the self-energy expression (44) that has to be used in equation (47). Putting everything together, we obtain a closed kinetic equation for the electron distribution function $f(\boldsymbol{k}) \equiv f^{<}(\boldsymbol{k})$

$$e\boldsymbol{E}\nabla_k f(\boldsymbol{k}) = \frac{2}{\hbar}\mathrm{Re}\sum_{q\lambda}\frac{|M_{q\lambda}|^2}{\sinh(\hbar\omega_{q\lambda}/2k_BT)}$$
$$\times\int_0^\infty \mathrm{d}t\,\exp\left\{\frac{\mathrm{i}}{\hbar}\int_0^t \mathrm{d}\tau\left[\varepsilon\left(\boldsymbol{k}-\frac{e\boldsymbol{E}}{\hbar}\tau\right)-\varepsilon\left(\boldsymbol{k}+\boldsymbol{q}-\frac{e\boldsymbol{E}}{\hbar}\tau\right)\right]\right\}$$

$$\times\left\{\cos\omega_{q\lambda}\left(t-\frac{\mathrm{i}\hbar}{2k_BT}\right)f\left(\boldsymbol{k}+\boldsymbol{q}-\frac{e\boldsymbol{E}}{\hbar}t\right)\left(1-f(\boldsymbol{k}-\frac{e\boldsymbol{E}}{\hbar}t)\right)\right.$$
$$\left.-\cos\omega_{q\lambda}\left(t+\frac{\mathrm{i}\hbar}{2k_BT}\right)f\left(\boldsymbol{k}-\frac{e\boldsymbol{E}}{\hbar}t\right)\left(1-f\left(\boldsymbol{k}+\boldsymbol{q}-\frac{e\boldsymbol{E}}{\hbar}t\right)\right)\right\}. \tag{49}$$

This quantum-kinetic equation applies to Fermions and reproduces exactly an equation, which is sometimes called the Barker–Ferry equation [19, 31–33, 45]. In the Boltzmann limit $(1-f\rightarrow 1)$, the known one-electron result is obtained, which has been derived and discussed many years ago [19, 22, 31, 32, 43, 45]. The kinetic equation (49) has been used [11, 46] to study miniband transport in SLs.

Appendix B

A kinetic equation for the distribution function $f(\boldsymbol{k}, t)$ is obtained from equations (44), and (45) together with (18). Taking into account that for a non-degenerate electron gas $G^{<}\rightarrow 0$, we get

$$-\tilde{G}^{>}(\boldsymbol{k},t)e\boldsymbol{E}\nabla_{\boldsymbol{k}}f\left(\boldsymbol{k}-\frac{e\boldsymbol{E}}{2\hbar}|t|\right)=\frac{2}{\hbar}\mathrm{Re}\sum_{q\lambda}\frac{|M_{q\lambda}|^2}{\sinh(\hbar\omega_{q\lambda}/2k_BT)}$$
$$\times\left\{\int_0^\infty \mathrm{d}t_1\left[\cos\omega_{q\lambda}\left(t-t_1+\frac{\mathrm{i}\hbar}{2k_BT}\right)\tilde{G}^{<}\left(\boldsymbol{k}+\boldsymbol{q}-\frac{e\boldsymbol{E}}{\hbar}t_1,t-t_1\right)\right.\right.$$
$$\times\tilde{G}^{>}\left(\boldsymbol{k}+\frac{e\boldsymbol{E}}{\hbar}(t-t_1),t_1\right)$$
$$-\cos\omega_{q\lambda}\left(t_1-\frac{\mathrm{i}\hbar}{2k_BT}\right)\tilde{G}^{>}\left(\boldsymbol{k}+\boldsymbol{q}+\frac{e\boldsymbol{E}}{\hbar}(t-t_1),t_1\right)$$
$$\left.\times\tilde{G}^{<}\left(\boldsymbol{k}-\frac{e\boldsymbol{E}}{\hbar}t_1,t-t_1\right)\right]$$
$$-\int_{-\infty}^0 \mathrm{d}t_1\left[\cos\omega_{q\lambda}\left(t_1-\frac{\mathrm{i}\hbar}{2k_BT}\right)\tilde{G}^{>}\left(\boldsymbol{k}+\boldsymbol{q}-\frac{e\boldsymbol{E}}{\hbar}(t-t_1),t_1\right)\right.$$
$$\times\tilde{G}^{<}\left(\boldsymbol{k}+\frac{e\boldsymbol{E}}{\hbar}t_1,t-t_1\right)$$
$$-\cos\omega_{q\lambda}\left(t-t_1+\frac{\mathrm{i}\hbar}{2k_BT}\right)\tilde{G}^{<}\left(\boldsymbol{k}+\boldsymbol{q}+\frac{e\boldsymbol{E}}{\hbar}t_1,t-t_1\right)$$
$$\left.\left.\times\tilde{G}^{>}\left(\boldsymbol{k}-\frac{e\boldsymbol{E}}{\hbar}(t-t_1),t_1\right)\right]\right\}. \tag{50}$$

$f(\boldsymbol{k}, t)$ enters the right-hand side of this equation via equation (18).

If the collisional broadening is small, the distribution function $f(\boldsymbol{k}, t)$ is effectively independent of t, and we get from equation (50) for $t = 0$

$$e\boldsymbol{E}\nabla_{\boldsymbol{k}}f(\boldsymbol{k})=\frac{2}{\hbar}\mathrm{Re}\sum_{q\lambda}\frac{|M_{q\lambda}|^2}{\sinh(\hbar\omega_{q\lambda}/2k_BT)}\int_0^\infty \mathrm{d}t\,\tilde{G}^{>}\left(\boldsymbol{k}+\boldsymbol{q}-\frac{e\boldsymbol{E}}{2\hbar}t,t\right)^*\tilde{G}^{>}\left(\boldsymbol{k}-\frac{e\boldsymbol{E}}{2\hbar}t,t\right)$$
$$\times\left\{\cos\omega_{q\lambda}\left(t-\frac{\mathrm{i}\hbar}{2k_BT}\right)f(\boldsymbol{k}+\boldsymbol{q}-e\boldsymbol{E}t/\hbar)\right.$$
$$\left.-\cos\omega_{q\lambda}\left(t+\frac{\mathrm{i}\hbar}{2k_BT}\right)f(\boldsymbol{k}-e\boldsymbol{E}t/\hbar)\right\}. \tag{51}$$

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